The occupancies of individual orbits and the nuclear matrix element of the ^{76}Ge neutrinoless $\beta\beta$ decay

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We discuss the variation of the nuclear matrix element (NME) for the neutrinoless double beta ($0\nu\beta\beta$) decay of ^{76}Ge when the wave functions are constrained to reproduce the experimental occupancies of the two nuclei involved in the transition. In the Interacting Shell Model description the value of the NME is enhanced about 15% compared to previous calculations, whereas in the QRPA the NME's are reduced by 20%-30%. This diminishes the discrepancies between both approaches. In addition, we discuss effect of the short range correlations on the NME in the light of the recently proposed parametrizations based on a consistent renormalization of the $0\nu\beta\beta$ transition operator.

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Very recently, there has been a strong experimental effort to extract the occupation numbers of the nuclei ^{76}Ge and ^{76}Se [1, 2] by accurate measurements of one nucleon transfer reactions. At present, both neutron and proton occupancies have been determined. The main motivation to study these nuclei is that they are the initial and final states of a $\beta\beta$ transition. Therefore, we have the possibility to compare these experimental results with the theoretical occupations and, if necessary, detect which modifications would be required in the effective interactions in order to obtain improved agreement with the experiment. In principle, this would lead to more reliable results when obtaining, for instance, the value of NME's for the $0\nu\beta\beta$ decay process.

In the case of the interacting shell model (ISM), the calculations reported so far [3, 4] were performed using the gcn28.50 interaction. This interaction was obtained by a global fit to the region comprised by the $1p_{3/2}$, $1p_{1/2}$, $0f_{5/2}$ and $0g_{9/2}$ orbits —r_3g valence space, r_3 standing for the $p = 3$ major oscillator shell, with corresponding energy $E = \hbar\omega\,(p + 3/2)$, except the highest j orbit. In addition, we had produced another interaction based on gcn28.50, aimed to improve locally the quadrupole properties of the nuclei in the $A = 76$ region. In particular this interaction makes ^{76}Se prolate as suggested by the experimental data. We denote it by rg.prolate.

When the experimental occupation numbers were published, we decided to compute them with the two available effective interactions, in order to check the stability of the ISM $0\nu\beta\beta$ NME's with respect to this property of the nuclear wave functions.

In Table I we compare the experimental occupancies along with the theoretical ones obtained with both the gcn28.50 and rg.prolate interactions. The occupancies obtained with the former are quite close to the experimental ones, specially in the case of ^{76}Ge. However, for ^{76}Se they lie somewhat further from experiment. On the contrary, the interaction rg.prolate produces occupancies for ^{76}Se which are almost perfect. The only drawback of this interaction is found on the proton occupancies in ^{76}Ge that slightly overfill the $0g_{9/2}$ orbit against the filling of the p orbits. In any case, the results obtained with both interactions compare reasonably well with the measured ones, while the rg.prolate interaction can be said to fit quite successfully the experimental numbers.

Table I: Proton and neutron occupation numbers of nuclei ^{76}Ge and ^{76}Se. Experiment from Refs. [1, 2] *vs* theoretical results, obtained for the gcn28.50 and rg.prolate interactions.

	$1p_{1/2}$+$1p_{3/2}$	$0f_{5/2}$	$0g_{9/2}$
		Neutrons	
^{76}Ge (exp)	4.87±0.20	4.56±0.40	6.48±0.30
^{76}Ge (gcn28.50)	5.19	5.02	5.79
^{76}Ge (rg.prolate)	4.83	4.78	6.39
^{76}Se (exp)	4.41±0.20	3.83±0.40	5.80±0.30
^{76}Se (gcn28.50)	4.86	4.54	4.60
^{76}Se (rg.prolate)	4.08	4.06	5.86
		Protons	
^{76}Ge (exp)	1.77±0.15	2.04±0.25	0.23±0.25
^{76}Ge (gcn28.50)	1.70	1.90	0.40
^{76}Ge (rg.prolate)	1.34	2.00	0.66
^{76}Se (exp)	2.08±0.15	3.16±0.25	0.84±0.25
^{76}Se (gcn28.50)	2.74	2.27	0.99
^{76}Se (rg.prolate)	2.12	2.79	1.08

The QRPA occupancies deviate more from measurements than our ISM values. In order to cure these discrepancies with the measured occupations, Suhonen *et al.* [5] and Šimkovic *et al.* [6] have adjusted the parameters of their reference Woods-Saxon potential in order to reproduce the experimental numbers. The former do it such as to obtain agreement at BCS level while the latter get the experimental numbers only after the QRPA correlations have been included.

In Figure 1 we have plotted the experimental occupancies compared to the theoretical ISM —gcn28.50 and rg.prolate— and QRPA values —both original and ad-

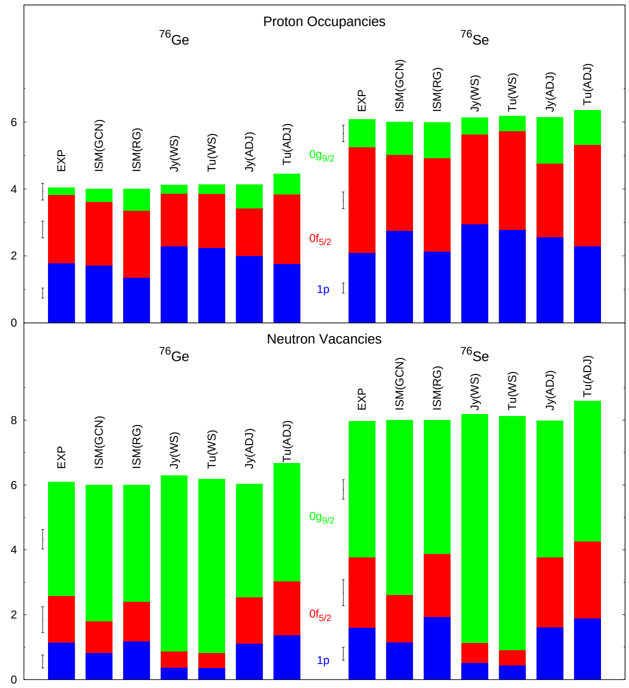

Figure 1: Comparison between experimental and theoretical occupation numbers for $A = 76$. Experimental values from Refs. [1, 2]. The ISM results correspond to the gcn28.50 (GCN) and rg.prolate (RG) interactions. The QRPA standard numbers, Tu(WS) and Jy(WS) give the occupancies at BCS level. The QRPA occupancies with adjusted single particle energies are given at BCS level in the case of Jy(ADJ) and at QRPA level for Tu(ADJ). Jy and Tu results from Refs. [5] and [6], respectively. The experimental error bars are also shown.

Table II: Values of the NME ($M^{0\nu\beta\beta}$) for the ^{76}Ge $\rightarrow$ ^{76}Se decay for ISM and QRPA calculations. QRPA(Jy)-WS and QRPA(Tu)-WS are the original QRPA calculations from Refs. [7] —for Jyväskylä— and [8] —for Tübingen. ADJ-WS are the calculations using a Woods-Saxon potential adjusted to reproduce the experimental occupancies, obtained from Refs. [5] (Jy) and [6] (Tu). UCOM type SRC's are considered. All results have $r_0 = 1.2$ fm and non-quenched axial coupling.

$M^{0\nu\beta\beta}$	GCN	WS	RG	ADJ-WS
ISM	2.81		3.26	
QRPA(Jy)		5.36		4.11
QRPA(Tu)		5.07-6.25		4.59-5.44

justed interactions for the Tübingen [6] and Jyväskylä [5] groups. We can observe that the amount of change in occupancies required to match the experiment is much larger in the case of the QRPA calculations, notably for neutrons. The effect of the new ISM interaction rg.prolate is much milder. In the end, all final interactions are able to reproduce the experimental occupations fairly well, with similar accuracies.

Once the interactions have been settled to give results as close as possible to experiment, the next step is to look at the NME's. In Table II we have collected their values for the ISM and QRPA with the six interactions considered, treating the SRC's by the UCOM prescription.

In the case of the Jyväskylä group, the NME suffers a substantial reduction of around 30% when calculated with the adjusted interaction. There is an effect in the same direction, whereas more moderate, present in the Tübingen's results. In this case, the reduction is closer to 20%. The different changes are probably related to the adjustment of experimental occupancies at BCS or QRPA level. These modifications can be traced back to the new values of the QRPA parameters g_{pp} achieved with the modified single particle energies, which are significantly different from those obtained with the original single particle energies, originated from Woods-Saxon potentials.

As for the ISM, the NME obtained with the rg.prolate interaction is enhanced with respect to the previously reported result obtained with gcn28.50. The increase is of some 15%. This means that our ISM is reasonably stable when obtained with different effective interactions. Moreover, when adjusting the interactions to agree with the measured occupancies in ^{76}Ge and ^{76}Se, the difference between the ISM and QRPA NME values diminishes. Notice however that expressing the effects in percentage may be misleading. Indeed, in the ISM case the NME increases by 0.45 while in the two QRPA calculations the reductions amount to 1.25 and 0.64, respectively.

The above analysis points out the relevance of occupation numbers in order to obtain a reliable result for the NME of the $0\nu\beta\beta$ decay. However, some caution needs to be taken regarding this point. For instance, we have observed that, performing calculations with truncations in the maximum seniority allowed in the wave functions (s_m), the occupancies obtained are essentially independent of s_m, while the NME is strongly reduced when high order seniority components are allowed in the wave functions. This can be observed in Table III. Therefore, it is concluded that occupation numbers by themselves do not fix the NME value, even though they are presumably necessary to get a sensible result.

In the same fashion, it is interesting to look at the variation of the nuclear matrix element of the $2\nu\beta\beta$ transition. Since the parameter g_{pp} is fixed in QRPA calculations in order to reproduce the experimental $2\nu\beta\beta$ matrix element, in their case no prediction is possible. On the contrary, within the ISM we can make this comparison. The result is that this matrix element is moderately enhanced as was the case of the $0\nu\beta\beta$ decay, changing from 0.32 MeV^{-1} obtained with the gcn28.50 interaction up to 0.41 MeV^{-1} when rg.prolate is employed. They are to be compared with the experimental number 0.14 ± 0.01 MeV^{-1} [9]. Beforehand, these theoretical values have to be quenched in order to take into account the valence

Table III: Occupancies and NME for the ^{76}Ge $\rightarrow$ ^{76}Se decay in function of the maximum seniority permitted in the wave functions, s_m. UCOM type SRC's.

	Neutrons			Protons			NME
	^{76}Ge						
	$1p$	$0f_{5/2}$	$0g_{9/2}$	$1p$	$0f_{5/2}$	$0g_{9/2}$	
$s_m = 0$	4.8	5.2	6.1	1.3	2.1	0.6	
$s_m = 4$	4.8	5.0	6.2	1.3	2.0	0.7	
$s_m = 10$	4.8	4.8	6.4	1.3	2.0	0.7	
	^{76}Se						
	$1p$	$0f_{5/2}$	$0g_{9/2}$	$1p$	$0f_{5/2}$	$0g_{9/2}$	
$s_m = 0$	3.9	4.6	5.5	1.8	3.3	0.9	11.85
$s_m = 4$	4.3	4.4	5.3	2.1	2.6	1.3	7.99
$s_m = 14$	4.1	4.1	5.9	2.1	2.8	1.1	3.26

Table IV: Values of the NME for the ^{76}Ge $\rightarrow$ ^{76}Se decay for ISM interactions, using the SRC's proposed in Ref. [10].

Interaction	$M_{no\,SRC}$	$M^{0\nu\beta\beta}_{Argonne}$	$M^{0\nu\beta\beta}_{Bonn}$
gcn28.50	2.89	2.82	3.00
rg.prolate	3.40	3.33	3.52

space truncation, which effectively quenches the Gamow-Teller strength. This quenching factor must lie between 0.7 for $0\hbar\omega$ spaces and 0.53 for the similar r_4h valence space. Taking 0.6 we get 0.12 and 0.15 MeV^{-1}, very close to the experiment.

Finally, we can study as well the NME of the same $0\nu\beta\beta$ decay in the light of very recent treatments of short range correlations (SRC) [10, 11]. These correlations haven been parametrized in the past by general prescriptions, but now, efforts are being made in order to study them consistently, this is, obtaining them from the regularization of the bare operator in the same way that the bare interaction is regularized into the effective one within the nuclear medium.

This is done in Refs. [10] and [11]. Both obtain similar results for the effect of short range correlations in the $0\nu\beta\beta$ process, which amount to a modification of less than 5% for the former and to a reduction of about 5% for the latter. If we compare this quantity to the one coming from the two standard parametrizations of SRC's for this decay, namely the Miller-Spencer parametrization of a Jastrow type function [12, 13] and the UCOM [14, 15] approach, it seems that the numbers obtained with the latter method are to be considered more accurate, being those coming from the Miller-Spencer parametrization an underestimation of the actual NME results.

Moreover, in Ref. [10] these effects are parametrized by two Jastrow type functions. Within the ISM we can take these two parametrizations and calculate the modification that they cause on the NME's. The results are shown on Table IV. They agree with those of Ref. [10], showing very mild modifications of the NME's by the SRC's, either a small increase —in the case of the parametrization that comes from the Bonn potential— or decrease —when the original potential is Argonne's.

In summary, we have calculated the orbital occupancies on nuclei ^{76}Ge and ^{76}Se with our previously used interaction gcn28.50 and the new one rg.prolate, and compared the results with the experimental numbers. In both cases the agreement is reasonable, but the results of the rg.prolate interaction agree extremely well with the experiment. When we compute the NME with the new interaction, the gcn28.50 value is enhanced some 15%, getting closer to QRPA values. At the same time, QRPA calculations reproducing successfully these experimental occupancies lower their previous NME's in about 20%, so overall, the gap between ISM and QRPA results is reduced nearly to one half of the previous value. This points out the importance of spectroscopic information in order to test the validity of the nuclear matrix elements of the neutrinoless double beta decay.

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